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# Testi del Syllabus

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Docente	<b>BONONI ALBERTO</b>	Matricola: <b>004854</b>
Anno offerta:	<b>2013/2014</b>	
Insegnamento:	<b>1005248 - DETECTION AND ESTIMATION</b>	
Corso di studio:	<b>5052 - COMMUNICATION ENGINEERING - INGEGNERIA DELLE TELECOMUNICAZIONI</b>	
Anno regolamento:	<b>2013</b>	
CFU:	<b>9</b>	
Settore:	<b>ING-INF/03</b>	
Tipo attività:	<b>B - Caratterizzante</b>	
Partizione studenti:	<b>-</b>	
Anno corso:	<b>1</b>	
Periodo:	<b>Primo Semestre</b>	

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# Testi in italiano

## Tipo testo

## Testo

### Lingua insegnamento

Inglese

### Contenuti

1. Detection Theory  
1.1 Bayes, MiniMax, Neyman Pearson Tests  
1.2 Multiple hypothesis testing MAP and ML tests  
1.3 Sufficient statistics  
Factorization, Irrelevance, Reversibility theorems  
1.4 MAP Test with Gaussian signals. Additive Gaussian noise channel  
1.5 Optimal detection of continuous-time signals: discrete representation. Orthonormal bases and signal coordinates. Gram-Schmidt procedure. Projection Theorem. Complete bases  
1.6 Discrete representation of a stochastic process. Karhunen Leove (KL) basis  
1.7 Optimal MAP receiver in AWGN  
1.8 Techniques to evaluate error probability  
1.9 Composite hypothesis testing: partially known signals in AWGN. Optimal incoherent MAP receiver structure  
1.10 Detection in additive colored Gaussian noise: whitening, Cholesky decomposition  
1.11 Detection with stochastic Gaussian signals: Radiometer

2. Estimation theory  
2.1 Fisherian estimation  
2.1.1 Minimum Variance Unbiased Estimation  
2.1.2 Cramer Rao Lower Bound  
2.1.3 Maximum Likelihood estimation  
2.2 Bayesian estimation  
2.2.1 Minimum Mean Square Error estimation  
2.2.2 MAP estimation  
2.2.3 Linear MMSE estimation  
2.2.4 Spectral Factorization and Wiener Filtering

### Testi di riferimento

Part I: Detection  
[1] J. Cioffi, "Signal Processing and Detection", Ch. 1, <http://www.stanford.edu/~cioffi>  
[2] B. Rimoldi, "Principles of digital Communications", EPFL, Lausanne. Ch 1-4.  
[3] A. Lapidoth, "A Foundation in Digital Communication" ETH, Zurich.  
[4] R. Raheli, G. Colavolpe, "Trasmissione numerica", Monte Universita' Parma Ed., Ch. 1-5. In Italian.

Part II: Estimation  
[5] S. M. Kay, "Fundamentals of statistical signal processing", Vol.1 (estimation), Prentice-Hall, 1998.

### Obiettivi formativi

L'obiettivo del corso è fornire allo studente la capacità di comprendere ed applicare le regole di base della teoria della decisione e della stima, e in particolare:

- i test statistici nel decidere tra diverse ipotesi
- la struttura del decisore ottimo nel contesto delle trasmissioni numeriche.
- i principali stimatori di uso comune
- la struttura dei filtri ottimi nel contesto delle trasmissioni numeriche.

Le capacità di applicare le conoscenze sopra elencate risultano essere in particolare:

- progettare ed analizzare le prestazioni del blocco di decisione nei ricevitori per trasmissioni numeriche
- progettare ed analizzare le prestazioni dei blocchi di stima dei parametri di segnale nei ricevitori per trasmissioni numeriche.

**Tipo testo****Testo****Prerequisiti**

Vedi testo inglese.

**Metodi didattici**

Lezioni teoriche per un totale di 63 ore ed esercitazioni per un totale di 9 ore.  
Esercizi assegnati per casa.

**Altre informazioni**

Vedi testo inglese

**Modalità di verifica dell'apprendimento**

Esami solo orali.  
Vedi dettagli nel testo inglese

**Programma esteso**

Vedi Testo Inglese



# Testi in inglese

## Tipo testo

## Testo

### Lingua insegnamento

Course Language

English

### Contenuti

1. Detection Theory  
1.1 Bayes, MiniMax, Neyman Pearson Tests  
1.2 Multiple hypothesis testing MAP and ML tests  
1.3 Sufficient statistics  
Factorization, Irrelevance, Reversibility theorems  
1.4 MAP Test with Gaussian signals. Additive Gaussian noise channel  
1.5 Optimal detection of continuous-time signals: discrete representation. Orthonormal bases and signal coordinates. Gram-Schmidt procedure. Projection Theorem. Complete bases  
1.6 Discrete representation of a stochastic process. Karhunen Leove (KL) basis  
1.7 Optimal MAP receiver in AWGN  
1.8 Techniques to evaluate error probability  
1.9 Composite hypothesis testing: partially known signals in AWGN. Optimal incoherent MAP receiver structure  
1.10 Detection in additive colored Gaussian noise: whitening, Cholesky decomposition  
1.11 Detection with stochastic Gaussian signals: Radiometer

2. Estimation theory  
2.1 Fisherian estimation  
2.1.1 Minimum Variance Unbiased Estimation  
2.1.2 Cramer Rao Lower Bound  
2.1.3 Maximum Likelihood estimation  
2.2 Bayesian estimation  
2.2.1 Minimum Mean Square Error estimation  
2.2.2 MAP estimation  
2.2.3 Linear MMSE estimation  
2.2.4 Spectral Factorization and Wiener Filtering

### Testi di riferimento

Reference Textbooks

Part I: Detection

[1] J. Cioffi, "Signal Processing and Detection", Ch. 1, <http://www.stanford.edu/~cioffi>

[2] B. Rimoldi, "Principles of digital Communications", EPFL, Lausanne. Ch 1-4.

[3] A. Lapidoth, "A Foundation in Digital Communication" ETH, Zurich.

[4] R. Raheli, G. Colavolpe, "Trasmissione numerica", Monte Universita' Parma Ed., Ch. 1-5. In Italian.

Part II: Estimation

[5] S. M. Kay, "Fundamentals of statistical signal processing", Vol.I (estimation), Prentice-Hall, 1998.

### Obiettivi formativi

Objective of the course is to provide the student with the ability to understand and apply the basic rules of detection and estimation theory, and in particular:

- to apply the most common statistical tests in deciding among different hypotheses
- to synthesize the structure of the optimal receiver and analyze its performance in the context of digital transmissions
- to apply the most common statistical estimators
- to synthesize the structure of the optimal filters and analyze their performance in the context of digital transmissions.

The abilities in applying the above-mentioned knowledge are in particular in the:

- design and performance analysis of the decision block in digital

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receivers  
- design and performance analysis of the parameter-estimation blocks in digital receivers

## Prerequisiti

Pre-requisites

Entry-level courses in probability theory and Fourier analysis for stochastic processes, such as those normally offered in the corresponding 3-year Laurea course, are necessary pre-requisites for this course.

## Metodi didattici

Teaching Methodology

Classroom teaching, 63 hours. In-class problem solving, 9 hours. Homeworks assigned weekly.

## Altre informazioni

Office Hours

Monday 11:30-13:30 (Scientific Complex, Building 2, floor 2, Room 2/19T).

## Modalità di verifica dell'apprendimento

Exams

Oral only, to be scheduled on an individual basis. When ready, please contact the instructor by email at [alberto.bononi@unipr.it](mailto:alberto.bononi@unipr.it) by specifying the requested date. The exam consists of solving some proposed exercises and explaining theoretical details connected with them, for a total time of about 1 hour. You can bring your summary of important formulas in an A4 sheet to consult if you so wish. Some sample exercises can be found on the course website. To get userid and password, please send an email to [alberto.bononi@unipr.it](mailto:alberto.bononi@unipr.it) from your account [nome@studenti.unipr.it](mailto:nome@studenti.unipr.it).

## Programma esteso

Syllabus (every class = 2 hours)

CLASS 1:

First hour: Course organization, objectives, textbooks, exam details. Sneaky preview of the course, motivations, applications. Second hour: basic probability theory refresher: total probability, Bayes rule in discrete/continuous/mixed versions, double conditioning. A first elementary exercise on binary hypothesis testing.

CLASS 2:

First hour: completion of proposed exercise. Second hour: Bayes Tests.

CLASS 3:

First hour: exercise on Bayes Test (Laplacian distributions) Second hour: MiniMax Test.

CLASS 4:

First hour: exercise on Minimax. Second hour: Neyman Pearson Test with example.

CLASS 5:

First hour: ROC properties. NP test with discrete RVs: randomization. Second hour: Exercise on Bayes, Minimax, Neyman-Pearson tests.

CLASS 6:

First hour: Multiple hypothesis testing, Bayesian approach. MAP and ML tests. Decision regions, boundaries among regions: examples in  $\mathbb{R}^1$  and  $\mathbb{R}^2$ . Second hour: exercise: 3 equally-likely signal "hypotheses"  $-A, 0, A$  in AWGN noise: Bayes rule (ML) based on the sample-mean (sufficient statistic).

CLASS 7:

First hour: Minimax in multiple hypotheses. Sufficient statistics: introduction. Second hour: Factorization theorem, irrelevance theorem. Reversibility theorem. Gaussian vectors refresher: joint PDF, MGF/CF.

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### CLASS 8:

First hour: Summary of known main results on Gaussian random vectors: Gaussian MGF, 4th order statistics from moment theorem, MGF-based proof of Gaussianity of linear transformations. Examples of Gaussian vectors: Fading Channel. Second hour: A: MAP Test with Gaussian signals. B: Additive Gaussian noise channel. Decision regions are hyperplanes.

### CLASS 9:

First hour: examples of decision regions. Optimal detection of continuous-time signals: motivation for their discrete representation. Second hour: Discrete signal representation: definitions. Inner product, norm, distance, linear independence. Orthonormal bases and signal coordinates.

### CLASS 10:

Gram-Schmidt orthonormalization. Detailed example. Operations on signals, and dual operations on signal images.

### CLASS 11:

Unitary matrices in change of basis. Orthogonal matrices: rotations and reflections. Orthogonality principle. Projection theorem. Interpretation of Gram-Schmidt procedure as repeated projections. Complete ON bases: motivations and definition.

### CLASS 12:

First hour: exercises: 1. product of unitary matrices is unitary. 2. unitary matrix preserves norm of vectors. Projection matrices, eigenvectors, eigenvalues, spectral decomposition. Properties. Second hour: examples of complete bases in  $L_2$ : the space of band-limited functions, evaluation of series coefficients, sampling theorem, ON check. More examples of complete bases: Legendre, Hermite, Laguerre.

### CLASS 13:

Discrete representation of a stochastic process. Mean and covariance of process coefficients. Properties of covariance matrices for finite random vectors: Hermiticity and related properties. Whitening. Karhunen Leove (KL) theorem for whitening of discrete process representation (hint to proof). Statement of Mercer theorem. KL bases.

### CLASS 14:

Summary of useful matrices: Normals and their subclasses: unitary, hermitian, skew-hermitian. If noise process is white, any ON complete basis is KL. Digital modulation. Example: QPSK. Digital demodulation with correlators bank or matched-filter bank.

### CLASS 15:

First hour: Matched filter properties. Max SNR, physical reason of peak at  $T$ . Second hour: back to M-ary hypothesis testing with time-continuous signals: receiver structure. With white noise, irrelevance of noise components outside signal basis. Optimal MAP receiver in AWGN. Basis detector. Signal detector.

### CLASS 16:

Examples of MAP RX and evaluation of symbol error probability  $P_e$ . First hour: MAP RX for QPSK signals and its  $P_e$ . Second hour: MAP RX for generic binary signals, basis detector, reduced complexity signal detector. Evaluation of  $P_e$ .

### CLASS 17:

First hour: Techniques to evaluate  $P_e$ : rotational invariance in AWGN and signal image shifts. Center of gravity for minimum energy. Second hour:  $P_e$  evaluation for binary signaling. Comparisons between antipodal and orthogonal signals. Calculation of  $P_e$  for 16-QAM (begin).

### CLASS 18:

First hour: Calculation of  $P_e$  for 16-QAM (end). Second hour: Calculation

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of  $P_e$  for M-ary orthogonal signaling. Begin calculation of Bit error rate (BER).

### CLASS 19:

Completion of BER evaluation in M-ary orthogonal signaling. Example: M-FSK. Occupied bandwidth. Limit as  $M \rightarrow \infty$  and connection with Shannon channel capacity. Notes on Simplex constellation. BER evaluation for QPSK: natural vs. Gray mapping.

### CLASS 20:

Further notes on Gray mapping. Approximate BER calculation: union upper bound, minimum distance bound, nearest-neighbor bound. Lower bounds. Example: M-PSK. Review of cartesian  $(X,Y)$ -to-polar  $(R,Q)$  probability transformation. For zero-mean normal  $(X,Y)$ ,  $(R,Q)$  are independent with Rayleigh and Uniform marginals.

### CLASS 21:

For non-zero-mean normal  $(X,Y)$ ,  $(R,Q)$  are dependent, with Rice and Bennet marginals. Properties of Rayleigh, Rice, Bennet PDFs. Use of Bennet PDF in the exact evaluation of  $P_e$  in M-PSK.

Composite hypothesis testing: introduction. Bayesian approach: Example of partially known signals in AWGN.

### CLASS 22:

Partially known signals in AWGN: Bayesian MAP decision rule. Application to incoherent reception of passband signals. Optimal incoherent MAP receiver structure.

### CLASS 23:

Alternative more compact derivation of incoherent MAP receiver for passband signals using complex envelopes. Incoherent OOK receiver and its BER evaluation.

### CLASS 24:

Detection in additive colored Gaussian noise. Karhunen-Loeve formulation. Hints about the analog whitening filter. Reversibility theorem and whitening of the discretized signal sample. Example 1: whitening by unitary transformation that aligns the orthonormal eigenvectors of the noise covariance matrix to the canonical basis. Example 2: Cholesky decomposition of covariance matrix and noise whitening. Example of calculation of Cholesky decomposition.

### CLASS 25:

Exercise: whitening and  $P_e$  evaluation for sampled signals in colored Gaussian noise.

Detection with stochastic signals: the case of Gaussian signals. Binary hypothesis testing: Radiometer. BER evaluation.

### CLASS 26:

Estimation theory: introduction. Classical (Fisherian) estimation. MSE cost. The bias-variance tradeoff. Example and motivation for unbiased estimators.

### CLASS 27:

Asymptotically unbiased and consistent estimators. MVUE. Cramer Rao Lower Bound: motivazione, theorem statement, example: signals in AWGN (both discrete and continuous-time). Amplitude estimation.

### CLASS 28:

Phase estimation. Proof of CRLB. Extension of CRLB to vector parameters: theorem statement and examples. ML estimation, introduction. If an efficient estimator exists, it is ML.

### CLASS 29:

ML: asymptotic properties and invariance. Examples: 1) Gaussian observations with unknown (constant) mean and variance. 2) Linear

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Gaussian model and comparison with least-squares solution. 3) Phase estimation of passband signals (begin)

CLASS 30:

ML: Phase estimation of passband signals (end). Bayesian Estimation: 1) MMSE estimator and minimum error. Orthogonality principle. Unbiasedness. Note on regression curve. Gaussian example. Exercise: both observations and parameter are negative exponentials.

CLASS 31:

Bayesian estimation: MAP estimator. Example. ML Criterion as a particular MAP case. Ex: linear Gaussian model (homework, with solution). Extension to vector parameters. Gaussian multivariate regression. MMSE linear Bayesian estimates. Optimal filter coefficients through orthogonality principle. Yule-Walker equations. LMMSE optimal estimator and minimal MSE.

CLASS 32:

Review of optimal scalar LMMSE estimator and minimum MSE. Extension to vector estimator. Wiener Filter: problem statement, objectives. A) Smoothing, optimal non-causal filter, MMSE error, case of additive noise channel. Alternative evaluation of MMSE with error filter.

CLASS 33:

B) Causal Wiener filter: problem setting in 2 steps: whitening and innovations estimation. Whitening: 1) review of two-sided Z-transform and its ROC. 2) review: Z-transform of PSD of the output of a linear system. 3) statement of Spectral Factorization (SF) theorem.

CLASS 34:

SF theorem: key to proof. Calculation of innovations filter  $L(z)$  for real processes through the SF. Regular processes classification with  $L(z)$  a rational fraction. AR, MA, ARMA processes. Example: AR(1).

CLASS 35:

Wiener causal filter, formula in  $z$ . Example.  $r$ -step predictor: form of filter in  $z$ . Error Formula.

CLASS 36:

Predictor: example: prediction of AR( $p$ ) processes.  $r$ -step filtering and prediction: formula in  $z$ . General error formula for additive noise channels. Example.